FIND THE FORM OF y_p FOR $y'' + 2y' = x^2(1 - e^{-2x})$

$= x^2 - x^2 e^{-2x}$

DON'T SOLVE FOR THE ACTUAL COEF'S

$r^2 + 2r = 0 \rightarrow r = 0, -2$

$y_h = c_1 + c_2 e^{-2x}$

x^2

$y_p = (Ax^2 + Bx + C)x$

$= Ax^3 + Bx^2 + Cx$

$x^2 e^{-2x}$

$y_p = Dx^2 e^{-2x} + Exe^{-2x} + Fe^{-2x}$

$= x(Dx^2 + Ex + F)e^{-2x}$

$= (Dx^3 + Ex^2 + Fx)e^{-2x}$

$= Dx^3 e^{-2x} + Ex^2 e^{-2x} + Fxe^{-2x}$

FORM OF $y_p = Ax^3 + Bx^2 + Cx + (Dx^3 + Ex^2 + Fx)e^{-2x}$

HIGHER ORDER LDE WITH CONSTANT COEF'S

SOLVE $y''' - 2y'' - 8y' = 0$

$$r^3 - 2r^2 - 8r = 0$$

$$r(r^2 - 2r - 8) = 0$$

$$r(r-4)(r+2) = 0 \longrightarrow r = 0, 4, -2$$

$$y = c_1 + c_2 e^{4x} + c_3 e^{-2x}$$

SOLVE $y''' + 3y'' + 28y' + 26y = 0$

$$r^3 + 3r^2 + 28r + 26 = 0$$

DESCARTES' RULE OF SIGNS

0 SIGN FLIPS → 0 POS REAL ROOTS

$(-r)^3 + 3(-r)^2 + 28(-r) + 26$

$-r^3 + 3r^2 - 28r + 26$

3 SIGN FLIPS → 3 or 1 NEG REAL ROOTS

RATIONAL ROOTS TH'M

POSSIBLE RATIONAL ROOTS

$$\pm \frac{\text{FACTOR OF CONSTANT TERM}}{\text{FACTOR OF LEADING COEF}}$$

$$r = \pm \frac{1, 2, 13, 26}{1}$$

$$= \pm 1, 2, 13, 26$$

TRY $r = -1, -2, -13, -26$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 28 & 26 \\ & & -1 & -2 & -26 \\ \hline & 1 & 2 & 26 & \boxed{0} \end{array}$$

0 ← REMAINDER

$r - -1$ IS A FACTOR

$(r+1)$ IS A FACTOR

$r^2 + 2r + 26$ IS THE OTHER FACTOR

$$(r+1)(r^2 + 2r + 26) = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 104}}{2}$$

$$r = -1 \pm 5i$$

$r = -1, -1 \pm 5i$

$$y = c_1 e^{-x} + c_2 e^{-x}\cos 5x + c_3 e^{-x}\sin 5x$$

SOLVE $\frac{d^4y}{dx^4} - y = 0$

$$y^{(4)} - y = 0 \longrightarrow r^4 - 1 = 0 \longrightarrow (r^2-1)(r^2+1) = 0$$

$$(r-1)(r+1)(r^2+1) = 0$$

$$r = 1, -1, \pm i$$

$$y = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$$

SOLVE $y^{(4)} + 2y'' + y = 0$

$$r^4 + 2r^2 + 1 = 0 \longrightarrow (r^2+1)^2 = 0 \longrightarrow r = \pm i, \pm i$$

$$y = c_1 \cos x + c_2 \sin x$$

$$+ c_3 x \cos x + c_4 x \sin x$$

$$= (c_3 x + c_1) \cos x + (c_4 x + c_2) \sin x$$

SOLVE $y''' + 3y'' + 3y' + y = 0$

$$r^3 + 3r^2 + 3r + 1 = 0 \longrightarrow (r+1)^3 = 0 \longrightarrow r = -1, -1, -1$$

$$y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$$

FIND THE FORM OF y_p FOR $y''' + 3y'' + 3y' + y = x^2e^{-x} + x^2$

$\hookrightarrow y_h = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$

$x^2 e^{-x}$

$y_p = (Dx^2e^{-x} + Exe^{-x} + Fe^{-x})x^3$

$= (Dx^3e^{-x} + Ex^2e^{-x} + Fxe^{-x})x^2$

$= (Dx^4e^{-x} + Ex^3e^{-x} + Fx^2e^{-x})x$

$= Dx^5e^{-x} + Ex^4e^{-x} + Fx^3e^{-x}$

$= (Dx^5 + Ex^4 + Fx^3)e^{-x}$

x^2

$y_p = Ax^2 + Bx + C$

FORM OF $y_p = Ax^2 + Bx + C + (Dx^5 + Ex^4 + Fx^3)e^{-x}$

4.6

NON-HOMOGENEOUS LDE WITH

VARIABLE COEFs OR FORCING TERMS NOT

ONLY INVOLVE SUMS/PRODUCTS

OF POLYNOMIALS, e^{kx},

$\cos bx$, $\sin bx$